Interaction of an electromagnetic wave with a suddenly stopped ionization front

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The theory of the interaction of an electromagnetic wave with a uniformly moving ionization front in a gas is extended to include the case when the front suddenly stops. This nonstationary character of the wave/front interaction, which is typical for experiments carried out in a finite-size gas tube, gives rise to fresh physical effects. First, currents induced near the plasma boundary after the front stops produce a static magnetic field not only in the plasma behind the front but also in the vacuum ahead of the front. Second, in the regime where the transmitted wave falls off behind the front, the skinning field leaks through the stopped front and produces a burst of highly frequency up-shifted radiation.

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I. INTRODUCTION

A strong laser pulse propagating in a gas can ionize the gas and thus can produce a gas/plasma boundary which moves with the group velocity of the ionizing pulse. Such a boundary is called an ionization front. An electromagnetic wave incident on the ionization front will be reflected and frequency up-shifted due to Doppler effect. The possibility of the generation of tunable frequency up-shifted radiation initiated the first theoretical works, which treated the interaction of an electromagnetic wave with an ionization front and were undertaken more than 30 years ago [1]. In the early 1990s, the emergence of lasers capable of producing fast ionization in gases brought in the subject for experimental tests and many theoretical predictions have been verified. In particular, the reflection of an electromagnetic wave from a moving ionization front and its transmission into the plasma were thoroughly studied [2]. It was found that the reflection from an underdense front yields very low energy efficiency of frequency up-conversion to warrant any practical use. Subsequently, it was proposed to use the transmitted wave which can also be highly frequency up-shifted and carry significant energy [3]. In experiments, however, the efficiency of up-conversion was rather poor [2]. One of possible reasons for the low energy conversion is the excitation of a self-sustaining distribution of dc currents and a static magnetic field in the plasma. Predicted more than 30 years ago [1], the experimental verification of the presence of this so-called free-streaming mode in the plasma still remains an open question. Later it was theoretically shown that the excitation of Langmuir waves behind the ionization front can also lead to significant losses for TM polarized waves [4]. Recently, the idea of frequency up-shifting was tested for optical frequencies by colliding two laser beams [5]. The prospects of using ionization fronts for terahertz generation in capacitor arrays filled with a gas or a semiconductor material are also currently discussed in the literature [6].

Traditionally, the theoretical works treated only the interaction of a wave with a uniformly moving front [1–4,7]. In experiments, however, the gas-filled tube has a finite length. When the ionizing pulse traverses the tube, it hits the quartz window at the end of the tube and the ionization front suddenly stops. This situation is typical, for example, for experiments on frequency up-shifting of electromagnetic radiation in a resonant microwave cavity [2]. Therefore, it is natural to extend the existing theories to include the case when the front suddenly stops and this is our aim in this paper. The nonstationary character of the wave/front interaction, caused by a sudden stop of the front, gives rise to several unusual effects. The first effect is the generation of a static magnetic field in vacuum ahead of the stopped front. The presence of dc currents and a static magnetic field was pointed out a long time ago [1]. However, in all the cases studied up to date, the static magnetic fields existed only in the plasma and not in vacuum. When the front stops, the dc currents excited behind the front will be modified by transient processes. The resultant distribution of the dc currents will give rise to a static magnetic field which exists not only in the plasma, as before the front stops, but also in vacuum. The presence of this field in vacuum may significantly simplify the experimental detection of the free-streaming mode. The second effect is the leakage of the transmitted wave into vacuum after the front stops. This is expected to occur when the transmitted wave follows the front. However, we show that the transmitted wave that initially falls off exponentially in the plasma behind the front can penetrate into vacuum and can produce a short burst of highly frequency up-shifted electromagnetic radiation. This opens the possibility to use the skinning transmitted wave as a source of frequency up-shifted radiation rather than using the propagating transmitted wave. Besides, the transmitted field behind the front has the maximum degree of frequency up-shift when the wave falls off in the plasma [4].

The paper is organized as follows. In Sec. II we lay out the theoretical model to descibe the interaction of an electromagnetic wave with an ionization front which suddenly comes to a stop. Section II contains a description of the electromagentic field distribution before the front stops, a procedure of finding the time evolution of the fields after the stop, and a discussion of main results. In Sec. III we investigate how a short pulse interacts with a moving front and the

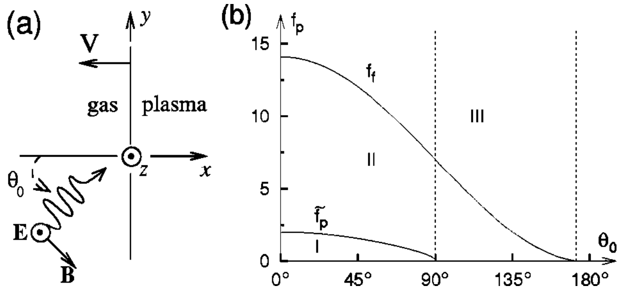

FIG. 1. (a) Schematic of a TE polarized wave incident on a moving ionization front. (b) Regimes of propagation for the transmitted wave in the plasma as a function of f_p and θ_0 for $\beta=0.99$ ($\theta_{\max}\approx 172°$). In region I [$f_p<\tilde{f}_p$ with $\tilde{f}_p=\sqrt{(\cos\theta_0+2\beta+\beta^2\cos\theta_0)\cos\theta_0}$], the transmitted wave propagates in the positive x direction; in region II ($\tilde{f}_p<f_p<f_f$), the transmitted wave runs after the front; in region III ($f_p>f_f$), the transmitted wave falls off exponentially behind the front.

relevance of stationary-state initial conditions for this situation. The final Sec. IV contains concluding remarks.

II. GENERAL THEORY

A. Initial conditions

We start by considering a plane TE polarized electromagnetic wave of frequency ω_0, which initially, at $t<0$, is incident at an angle θ_0 on an ionization front which moves with velocity $V(V<c$, with c the *in vacuo* speed of light) in the negative x direction [see Fig. 1(a)]. We assume the front to be infinitely sharp, i.e., the distance at which the plasma density increases is small compared with the wavelength of the incident, as well as reflected and transmitted, waves. Such fronts can nowadays be routinely produced by femtosecond laser pulses [8]. This approximation allows us to write the plasma density as $N(x,t)=N\Theta(x+Vt)$, where $\Theta(x+Vt)$ is the Heavyside step function. The complex electric field of the incident wave is

$$\mathbf{E}(x,y,t)=\hat{\mathbf{z}}E_0\exp(i\omega_0 t-ig_0x-ih_0y) \tag{1}$$

with $g_0=(\omega_0/c)\cos\theta_0$ and $h_0=(\omega_0/c)\sin\theta_0$ being the normal and tangential wave-vector components, respectively. Due to the translational invariance in the y direction, all fields have the same spatial dependence along the y direction and the common factor $\exp(-ih_0y)$ will be omitted from now on. The angle of incidence θ_0 lies between 0 and $\theta_{\max}$ with $\cos\theta_{\max}=-\beta$, $\beta=V/c$. The incident wave gives rise to reflected and transmitted waves, and a free-streaming mode. The continuity of the phase across the front yields the frequencies and the angles of propagations of these waves [4]. The frequencies of the reflected and transmitted waves are

$$f_r=\omega_r/\omega_0=\gamma^2(1+2\beta\cos\theta_0+\beta^2), \tag{2}$$

$$f_t=\omega_t/\omega_0=\gamma^2 f-\beta\gamma\sqrt{f_f^2-f_p^2}, \tag{3}$$

where $\gamma=1/\sqrt{1-\beta^2}$, $f=1+\beta\cos\theta_0$, $f_f=\gamma(\beta+\cos\theta_0)$, $f_p=\omega_p/\omega_0$, where $\omega_p=\sqrt{4\pi Ne^2/m}$ is the plasma frequency (e and m are the electron charge and mass, respectively), and for the free-streaming mode $f_s=\omega_s/\omega_0=0$. The angles of propagations are $\tan\theta_{r,t,s}=h_0/g_{r,t,s}$ with $g_{r,t,s}=\omega_0(f-f_{r,t,s})/V$ being the normal components of the wave vectors. The transmitted wave behaves quite unusually compared to the case of a stationary boundary. Depending on the plasma density, the angle of incidence, and the velocity of the front, three possible cases shown in Fig. 1(b) exist: the wave can propagate away from the front, follow the front, or fall off exponentially behind the front. The amplitudes of the excited waves are found using the continuity condition for the electric and magnetic fields and their spatial and temporal derivatives [4]:

$$E_r=E_0(f_t-1)/(f_r-f_t), \tag{4}$$

$$E_t=E_0(f_r-1)/(f_r-f_t), \tag{5}$$

$$B_{sy}=-g_sB_{sx}/h_0=-E_0f\beta^{-1}(1-f_r^{-1})(1-f_t^{-1}). \tag{6}$$

The frequencies, the angles of propagations, and the amplitudes completely determine the stationary picture of the wave transformation at a uniformly moving front before it comes to a stop.

B. Solution of initial value problem

We assume that at $t=0$ the front suddenly stops. In practice, the sudden stop means that it is much shorter than the periods of the waves. This condition is satisfied if the ionizing pulse reaches the boundary of the homogeneous gas volume bound by a glass or any other material. For simplicity, we assume a sharp vacuum/plasma boundary. We are interested in finding what happens with the fields and the currents in the plasma. To solve this nonstationary problem we will use Maxwell's equations

$$ih_0E_z=\frac{1}{c}\frac{\partial B_x}{\partial t}, \tag{7}$$

$$\frac{\partial E_z}{\partial x}=\frac{1}{c}\frac{\partial B_y}{\partial t}, \tag{8}$$

$$\frac{\partial B_y}{\partial x}+ih_0B_x=\frac{1}{c}\frac{\partial E_z}{\partial t}-\frac{4\pi}{c}eNv_z[\Theta(x+Vt)\Theta(-t)+\Theta(x)\Theta(t)] \tag{9}$$

supplemented by the equation for electron motion:

$$\frac{\partial v_z}{\partial t}=-\frac{e}{m}E_z. \tag{10}$$

To solve Eqs. (7)–(10) we will use a Laplace transform technique. The initial conditions for the fields and electron velocities at $t=0^+$ are the same as for $t=0^-$, since the fields and velocities do not change during the infinitely short stopping time. From Eqs. (7)–(10) we obtain the equation for the Laplace transform $\mathcal{E}(s)$ of the electric field $E_z(t)$:

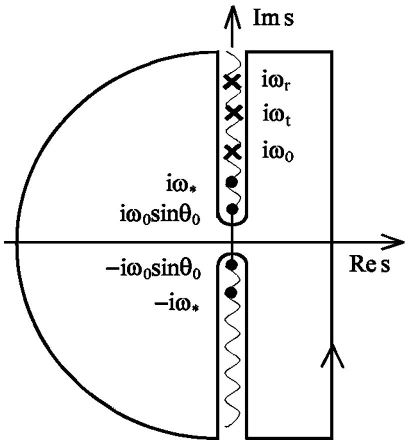

FIG. 2. Integration contour for $t>|x|/c$.

$$\frac{\partial^2 \mathcal{E}}{\partial x^2}-\left(\frac{s^2}{c^2}\varepsilon+h_0^2\right)\mathcal{E}=F(x,s), \tag{11}$$

where $\varepsilon=1+\omega_p^2/s^2$ in the plasma and $\varepsilon=1$ in vacuum. The source term $F(x,s)$ in Eq. (11) is defined by the initial fields in the plasma, $F(x,s)=-(1/c^2)(s+i\omega_t)E_t\exp(-ig_t x)$ and in vacuum, $F(x,s)=-(1/c^2)[(s+i\omega_0)E_0\exp(-ig_0x)+(s+i\omega_r)E_r\exp(-ig_r x)]$. Solving Eq. (11) in the homogeneous regions ($x<0$ and $x>0$) and matching the solutions by the boundary conditions (continuity of $\mathcal{E}$ and $\partial\mathcal{E}/\partial x$), we find

$$\mathcal{E}(x,s)=\begin{cases}\dfrac{E_0e^{-ig_0x}}{s-i\omega_0}+\dfrac{E_re^{-ig_rx}}{s-i\omega_r}+A_v(s)e^{\kappa_v x} & \text{if } x<0\\[2ex] \dfrac{E_te^{-ig_tx}}{s-i\omega_t}+A_p(s)e^{-\kappa_p x} & \text{if } x>0,\end{cases} \tag{12}$$

where

$$A_{v,p}(s)=\mp\frac{1}{\kappa_v+\kappa_p}\left(\frac{\kappa_{p,v}\mp ig_0}{s-i\omega_0}E_0+\frac{\kappa_{p,v}\mp ig_r}{s-i\omega_r}E_r-\frac{\kappa_{p,v}\mp ig_t}{s-i\omega_t}E_t\right) \tag{13}$$

and $\kappa_{p,v}=\sqrt{\varepsilon s^2/c^2+h_0^2}$.

Expressions (12) and (13) give the solution of the problem in the s domain. To obtain the solutions in the time domain we have to take the inverse Laplace transform. The first terms in Eq. (12) describe the forced responses and contribute to $E_z(x,y,t)$ immediately at $t=0^+$. In vacuum, the poles of the two forced terms give the incident wave and the initially reflected wave; in the plasma, there is one pole which gives the initially transmitted wave. The free-streaming mode does not have any electric field and cannot be obtained directly from Eq. (12). At an arbitrary point x, the effect of the stopped front emerges at $t=|x|/c$. From this moment the free-wave terms in Eq. (12) start to contribute. In taking the inverse transform for $t>|x|/c$, we choose the integration contour (see Fig. 2) in the Riemann sheet in the complex s plane, where the real parts of $\kappa_{p,v}(s)$ are positive to ensure evanescence of the fields at $x\to\pm\infty$. The branch cuts due to double-valued functions $\kappa_{p,v}(s)$ run along the imaginary axis from the branch points $\pm i\omega_0\sin\theta_0$ and $\pm i\omega_*$ ($\omega_*=\sqrt{\omega_p^2+\omega_0^2\sin^2\theta_0}$) to infinity.

The electric field distribution formed at $t\to+\infty$ is given by the poles of the forced responses and the integrals of $A_{v,p}(s)$ along the right-hand side of the branch cut near the poles $s=i\omega_{0,r,t}$. This asymptotic distribution depends on whether the front and the initial wave were copropagating or counterpropagating at $t<0$. For the counterpropagating case, the initial wave will produce the textbook reflected and transmitted waves due to incidence upon the motionless plasma boundary. This does not happen in the copropagating case. Besides the incident wave, the transmitted wave, if at $t<0$ it followed the front, will contribute to the asymtotic distribution due to its reflection and transmission at the boundary.

C. Free-streaming mode

We now focus on the excitation of the free-streaming mode and then discuss transient radiation into vacuum. The magnetic field can be found from Eqs. (12) and Maxwell's equation (in the s domain): $\mathcal{B}_x=[ich_0\mathcal{E}+B_x(t=0)]/s$. The pole at $s=0$ gives the desired static magnetic field:

$$B_x^{\rm st}(x)=B_0^{\rm st}\begin{cases}(B_1+B_2)e^{h_0x} & \text{if}\quad x<0\\ B_1e^{-ig_sx}+B_2e^{-\kappa_p^{\rm st}x} & \text{if}\quad x>0,\end{cases} \tag{14}$$

where the complex amplitude $B_0^{\rm st}$ is

$$B_0^{\rm st}=E_0\sin\theta_0(1-f_r^{-1})(1-f_t^{-1})/B_1, \tag{15}$$

where $B_2=-(\sin\theta_0+if/\beta)/C$, $B_1=(\sin\theta_0+f_*)/C$, $C=\sqrt{f_*^2+f^2/\beta^2}$, $|B_1+B_2|=1$, $\kappa_p^{\rm st}=f_*\omega_0/c$, and $f_*=\omega_*/\omega_0$. $B_y^{\rm st}$ can be found by combining Maxwell's equation $\nabla\cdot\mathbf{B}=0$ and Eq. (14). The static magnetic field in the plasma consists of two terms. The first term describes the usual field formed behind the uniformly moving front. The second term results from the transient processes near the boundary after the front stops. Since at $x\gg1/\kappa_p^{\rm st}$ the second term vanishes, the stopping of the front causes a change in the static field only near the boundary at the depth $\sim1/\kappa_p^{\rm st}$. A striking effect, though, is the excitation of a static magnetic field in vacuum [see Eq. (14)]. The lines of $\mathbf{B}^{\rm st}$ in vacuum continue the lines existing in the plasma [see Fig. 3(a)]. The two components $B_{x,y}^{\rm st}$ change harmonically along the y axis; they have the same amplitude but shifted in phase by $\pi/2$. The magnitude of $\mathbf{B}^{\rm st}$ falls off exponentially with the distance from the boundary; in vacuum the characteristic decay length is h_0^{-1}. For $\sin\theta_0\sim1$ this decay length is comparable to the wavelength of the incident wave, which lies usually in the centimeter range in experiments on frequency up-shifting of microwave radiation. For small values of $\sin\theta_0$, i.e., for the angles of incidence θ_0 close to 0 ° or 180 °, the decay length can significantly exceed the wavelength of the initial wave. This circumstance may simplify the experimental demonstration of the existence of the free-streaming mode. However, when θ_0 approaches 0 ° or 180 ° the amplitude of the mag-

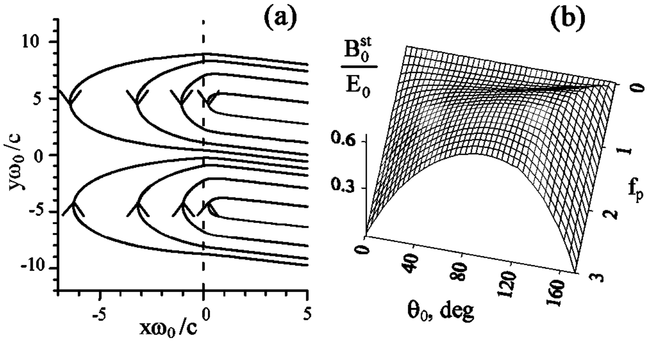

FIG. 3. (a) Lines of the static magnetic field for $\beta=0.99$, $f_p=3$, and $\theta_0=160°$. (b) Amplitude of the constant magnetic field at the plasma boundary for $\beta=0.99$.

netic field approaches zero [see Fig. 3(b)]. Nevertheless, there are regions of angles where the decay length is large and the amplitude of the magnetic field is also significant. In addition, the amplitude of the static magnetic field increases with the plasma density.

The static magnetic field discussed above is sustained by dc currents in the plasma. These currents can be obtained from $\mathbf{j}^{st}=(c/4\pi)\nabla\times\mathbf{B}^{st}$, which gives

$$j_z^{st}(x)=\frac{i\omega_0 B_0^{st}}{4\pi\sin\theta_0}\left(\frac{f_r B_1}{\gamma^2\beta^2}e^{-ig_s x}-f_p^2 B_2 e^{-\kappa_p^{st}x}\right). \quad (16)$$

A typical distribution of the dc currents is shown in Fig. 4(a). Similar to the static magnetic field, the dc currents become distorted near the boundary. The depth of this distortion is the same as for the static magnetic field.

In the analysis above we neglected collisions. The inclusion of collisions would lead to a gradual damping of the dc currents and, consequently, to a finite lifetime of the static magnetic field. For gas pressures typically used in experiments [2,6] this lifetime is $\sim$10–100 ns and, thus, may significantly exceed both the duration of the incident pulse with wavelength $\lambda_0\sim1$ cm and time scale of the transient electromagnetic processes.

D. Reradiation of the skinning field

We now turn to the radiation from the plasma after the front stop. In general, the transient processes are described by the integrals along the branch cuts (see Fig. 2). The outgoing radiation (going from the boundary to $x\to\pm\infty$) is described by the integrals along the right-hand sides of the branch cuts in the intervals $\omega_0\sin\theta_0<|\omega|<\infty$ for $x<0$ and $\omega_*<\omega<\infty$ for $x>0$ with $\omega=\mathrm{Im}(s)$. These integrals give the expansion of the radiation into outgoing plane waves with different frequencies related to the angle of propagation θ (measured in the x-y plane from the normal to the boundary direction both in the plasma and vacuum) by

$$\omega=\begin{cases}\dfrac{\sin\theta_0}{\sin\theta}\omega_0 & \text{if } x<0\\[2ex] \dfrac{\omega_*}{\sin\theta}\sqrt{1-\dfrac{\omega_p^2}{\omega_*^2}\cos^2\theta} & \text{if } x>0.\end{cases} \quad (17)$$

The closer the angle θ to the normal, the higher the frequency of the wave. The angular density of energy radiated into the vacuum and plasma is [9]

$$\mathcal{W}_{v,p}(\theta)=\frac{c^2h_0\cot^2\theta}{16\pi^2}|A_{v,p}(s=i\omega)|^2. \quad (18)$$

The formula (18) is valid for all angles except the angles at which the forced responses in Eq. (12) contribute. The energy emitted at these angles is infinite due to our assumption of a plane incident wave. Figure 4(b) shows the angular density of the radiation generated in vacuum in the case when the initial wave falls off behind the moving front. For the parameters we used, the decay depth of the wave is about three periods. After the front stops, this exponentially decaying field generates a wave packet propagating at 1.2° in vacuum. Integration of $\mathcal{W}_v(\theta)$ over the interval around 1.2° gives the energy which is almost equal to the energy of the skinning transmitted wave. This means that the skinning wave leaks into vacuum after the front stops and produces a burst of radiation at frequency up-shifted by a factor of $\mathrm{Re}\,f_t\approx8.1$ compared to ω_0. The reflection of this wave into the plasma is negligible. By changing the degree of skinning one can control the duration of the wave packet emitted into vacuum. The initially reflected field propagates at $\theta\approx0.65°$ [see Fig. 4(b)] with frequency $f_r\approx15$ and amplitude $\sim E_0$. Despite the high degree of frequency up-shift and large amplitude, it carries a small fraction of the incident energy even in the case of a uniformly moving front because the length of the wave packet is very small [4]. In the copropagating case and for $\beta\approx1$, it takes a long time for the reflected wave packet to separate from the front. In a finite-size gas tube, the reflected wave packet may not even form before the front reaches the end of the working volume. However, as we showed, even the decaying transmitted field can escape the plasma volume and produce frequency up-shifted radiation.

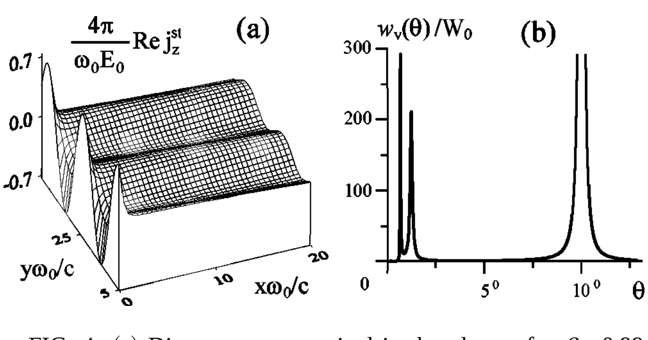

FIG. 4. (a) Direct currents excited in the plasma for $\beta=0.99$, $f_p=3$, and $\theta_0=160°$. (b) Angular density of the radiation in vacuum $\mathcal{W}_v(\theta)/W_0$ [normalized to the quantity $W_0=E_0^2c/(4\omega_0)$] for $\beta=0.999$, $\theta_0=170°$, and $f_p=0.318$.

III. INTERACTION OF A SHORT PULSE WITH A MOVING FRONT IN A WAVEGUIDE

The developed theory is based on the assumption that a steady-state distribution of the electromagnetic field is formed before the front stops. From the experimental point

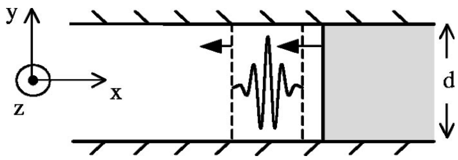

FIG. 5. Schematic of a pulse overtaken by an ionization front in a waveguide. The pulse shown corresponds to the spatial electric field dependence used in calculations.

of view, this assumption requires taking a cw wave (or a very long pulse) and letting the front interact with it. Once a steady state is formed the front should stop before the incident pulse is completely overtaken by the front. Usually experiments are carried out in finite-length waveguides or gas chambers, and very often short pulses are used rather than cw fields. This requires a critical examination of relevance of the steady-state initial conditions described in Sec. II A to typical experimental situations. Our purpose in this section is twofold. First, we study how a short pulse interacts with a uniformly moving ionization front which overtakes the pulse. Second, we show that before a reflected pulse is formed, a significant portion of the energy of the pulse is localized behind the front in evanescent field. Thus, if the front stops at this moment the evanescent field produces a burst of radiation into vacuum ahead of the front. Since our primary aim in this section is to investigate the time dependence of the electromagnetic field distribution induced near the plasma boundary by a short pulse, we do not model the stopping of the front.

To study the interaction of a short pulse with the ionization front, we chose the case when the pulse propagates in a waveguide rather than in free space (see Fig. 5). The waveguide is formed by two perfectly conducting planes at $y=\pm d/2$. The interaction in such a waveguide can be treated as a special case of oblique incidence. We assume that the incident pulse is Gaussian and it consists of the components of the lowest TE mode of the waveguide. The electric field of such a pulse has only a z component, which can be expanded into plane waves with different frequencies and longitudinal components of the wave vector:

$$E_z^{(\mathrm{i})}=\widetilde{E}_0\int_{-\infty}^{\infty}d\omega e^{-(\omega-\omega_0)^2\tau^2/2+i\omega t-igx}, \quad (19)$$

where $\widetilde{E}_0=E_0\cos(\pi y/d)(\tau/\sqrt{2\pi})$, τ is the duration of the pulse, ω_0 is the central frequency, $g=-(\omega/c)\sqrt{1-(c\pi/\omega d)^2}$. The magnetic field of the incident pulse can be found by substituting Eq. (19) into Maxwell's equations. The reflected and transmitted pulses can be found by multiplying each Fourier component of the incident pulse by the reflection and transmission coefficients given by Eqs. (4) and (5). This gives the reflected pulse

$$E_z^{(\mathrm{r})}=\widetilde{E}_0\int_{-\infty}^{\infty}d\omega\frac{f_t-1}{f_r-f_t}e^{-(\omega-\omega_0)^2\tau^2/2+i\omega_r t-ig_r x} \quad (20)$$

and the transmitted pulse

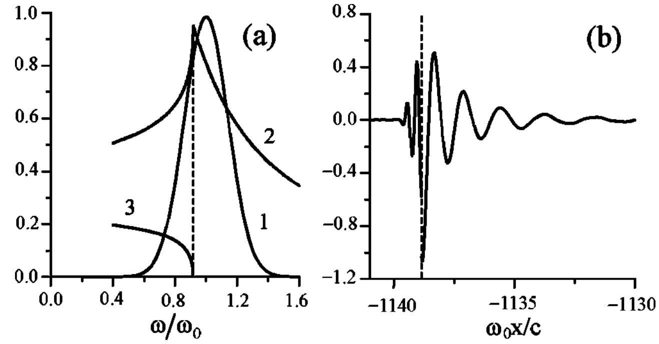

FIG. 6. (a) Energy spectrum (in arbitrary units) of the incident pulse (1) together with the frequency conversion coefficient $0.1\,\mathrm{Re}\,f_t$ [given by Eq. (3)] of the transmitted wave (2) and the coefficient of the energy conversion of a quasi-monochromatic incident wave into the propagating transmitted wave (3). (b) Reflected (for $\omega_0 x/c<-1138.86$) and transmitted (for $\omega_0 x/c>-1138.86$) electric field $E_z^{(r,t)}/E_0$ at $\omega_0 t=1140$ and $y=0$. The position of the front is indicated by the dashed line.

$$E_z^{(\mathrm{t})}=\widetilde{E}_0\int_{-\infty}^{\infty}d\omega\frac{f_r-1}{f_r-f_t}e^{-(\omega-\omega_0)^2\tau^2/2+i\omega_t t-ig_t x}. \quad (21)$$

The y component of the constant magnetic field excited in the plasma is

$$B_y^{(\mathrm{s})}=-\widetilde{E}_0\frac{1}{\beta}\int_{-\infty}^{\infty}d\omega f(1-f_r^{-1})(1-f_t^{-1})e^{-(\omega-\omega_0)^2\tau^2/2-ig_s x} \quad (22)$$

and the x component is found from $\nabla\cdot\mathbf{B}=0$. The wave vectors $g_{r,t,s}$ and frequencies $f_{r,t}$ in Eqs. (20)–(22) are the same as for an incident plane wave and they are given in Sec. II A. Since the frequency ω_t may be complex, the transmitted wave (21) describes both the waves that propagate and decay in the plasma. The values of the reflected and transmitted fields as well as of the constant magnetic field allow finding the energy of the corresponding modes.

We chose the parameters of the pulse such that the excitation of the propagating waves in the plasma is rather small: $f_p=0.35$, $\omega_0\tau=5$. The actual form of the pulse is shown in Fig. 5. The width of the waveguide is chosen to provide an angle of incidence of 170° for the central component of the pulse and front's velocity $\beta=0.999$. Figure 6 shows the energy spectrum of this pulse and the energy conversion coefficient into the propagating transmitted wave. Most of the energy of the incident pulse lies in the region where no propagating waves can be excited, and thus a significant excitation of the skinning field is expected. This choice of parameters allows also to obtain high-frequency conversion since the frequency of the transmitted field (see Fig. 6) reaches a maximum value when the transition from the propagating regime to the skinning regime occurs.

Figure 7 shows the energy of different modes excited by the pulse as a function of time. Most of the energy of the incident pulse goes into the excitation of the free-streaming mode in the plasma. The reflected energy grows rather slowly in time while the energy accumulated in the plasma

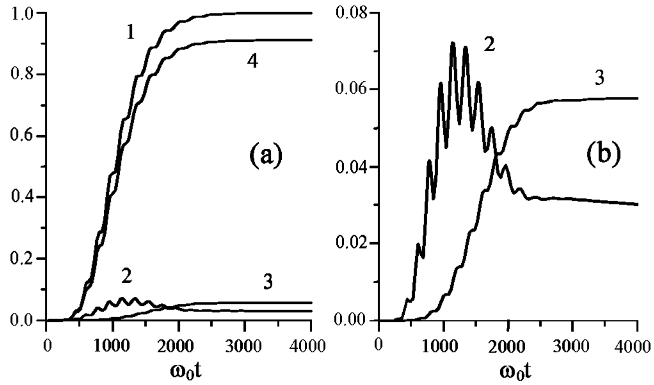

FIG. 7. (a) Energy of the incident pulse that was overtaken by the ionization front (1), energy of the transverse modes in the plasma (2), energy of the reflected pulse (3), and energy of the free-streaming mode (4). (b) Same as (a) but only curves (2) and (3) are shown. All energies are normalized to the total energy of the incident pulse.

grows much faster. After some time, the front overtakes most of the pulse and the energy accumulated in the plasma starts to decrease due to its reradiation into the reflected pulse. The delay in the reflection can be explained by partial penetration of the pulse into the plasma in the form of a decaying field. Thus, we can conclude that the formation of the decaying wave requires less time compared to the formation of the reflected pulse. The significant delay in the formation of the reflected pulse can be crucial in experimental situations where the interaction time is limited by the length of the waveguide. Fortunately, even if the front stops at the moment when the reflected pulse is not formed yet but there is already a significant fraction of the energy accumulated behind the front, the accumulated energy can escape the plasma and produce the frequency up-shifted radiation, as we showed in Sec. II. The electric field distribution at such a moment is shown in Fig. 6(b).

IV. CONCLUSION

To conclude, we have shown that the interaction of an electromagnetic wave with a suddenly stopped ionization front gives rise to two nontrivial physical effects: first, generation of a static magnetic field in vacuum ahead of the front, and second, emission of a highly frequency up-shifted skinning field in the plasma into vacuum. We also showed that even if the incident pulse is very short, it can produce a significant skinning field much earlier compared to the formation of the reflected pulse. Stopping the front at such a moment should give rise to reradiation of the skinning energy as in the case of a cw incident field. While the generation of the static field exists only for a TE polarized incident wave, the reemission of the skinning field, in general, takes place for an arbitrary polarization of the incident field. Despite the possibility to obtain highly frequency up-shifted transients both for TE and TM incident waves, the energy of the transients will be different in these cases, since the TM wave can excite Langmuir waves behind the front [4].

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[1] V. I. Semenova, Sov. Radiophys. Quantum. Electron. **10**, 599 (1967); **15**, 505 (1972).

[2] R. L. Savage, Jr. *et al.*, Phys. Rev. Lett. **68**, 946 (1992); IEEE Trans. Plasma Sci. **21**, 5 (1993).

[3] W. B. Mori, Phys. Rev. A **44**, 5118 (1991).

[4] M. I. Bakunov and A. V. Maslov, IEEE Trans. Plasma Sci. **27**, 655 (1999); M. I. Bakunov *et al.*, Phys. Rev. E **63**, 037401 (2001).

[5] J. M. Dias *et al.*, Phys. Rev. Lett. **78**, 4773 (1997).

[6] W. B. Mori *et al.*, Phys. Rev. Lett. **74**, 542 (1995); C. H. Lai *et al.*, *ibid.* **77**, 4764 (1996); P. Muggli *et al.*, Phys. Plasmas **5**, 2112 (1998); T. Higashiguchi *et al.*, Phys. Rev. Lett. **85**, 4542 (2000); D. Hashimshony *et al.*, *ibid.* **86**, 2806 (2001).

[7] M. Lampe *et al.*, Phys. Fluids **10**, 42 (1978).

[8] T. Brabec and F. Krausz, Rev. Mod. Phys. **72**, 545 (2000).

[9] M. I. Bakunov and A. V. Maslov, Phys. Rev. E **57**, 5978 (1998).